Engineering Notes

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Numerical Evaluation of Transonic Wave Drag

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Introduction

THE wave drag coefficient for a pointed slender body of length L is given, in terms of its cross-sectional area distribution $\mathcal{S}(x)$, by[1]

$$AR \cdot C_{D_W} = -\frac{L^2}{2\pi}\int_0^1\int_0^1 S''(\xi)S''(\zeta)\ln|\xi-\zeta|\,d\xi d\zeta + T \quad (1)$$

where

$$T=\frac{L^2}{2\pi}[S'(1)]^2\ln[2/\lambda r(1)] + \frac{L^2}{2\pi}S'(1)\int_0^1 S''(\xi)\ln(1-\xi)d\xi \Big| \quad (1a)$$

Here $S(\xi)=\mathcal{S}(x)/L^2$, $\xi=\zeta=x/L$, $r(1)=R(L)/L$, $\lambda=(M^2-1)^{1/2}$, and AR denote the reference area, $R(L)$ the base radius, M the Mach number, and the primes denote differentiation. It should be noted that in the derivation of Eq. (1) it is implicit that $\mathcal{S}'(0)=0$ and that $\mathcal{S}'(x)$ is continuous in $0\leq x\leq L$. More involved formulas are required when $\mathcal{S}'(x)$ has a finite number of discontinuities.[2]

It must be emphasized that Eq. (1) is derived on the basis of linearization (and also that of a slender body) and therefore is not valid for the transonic regime where linearization is indeed questionable. In fact, Eq. (1) gives infinite drag at $M=1.0$ for bodies with $\mathcal{S}'(L)\neq 0$. However, for bodies with $\mathcal{S}'(L)=0$, Eq. (1) becomes independent of Mach number in which case only the double integral remains. Because of this Mach number independence, one is inclined to extend, for bodies with $\mathcal{S}'(L)=0$, the range of validity of the equation and consider the double integral to give the wave drag for all $M\geq 1$. Experience shows that such an extension does give reasonable wave drag estimates of engineering value.

Among the number of numerical procedures that are available for evaluation of the double integral for a given discrete point data set of values of $\mathcal{S}(x)$, those of Eminton[3] and Shanbhag and Narasimha[4,5] are well-known. However, their accuracy has only been assessed for smooth distributions allowing an analytical description. Such an assessment can be very misleading since the accuracy for a distribution representing an aircraft is dependent, not only on the shape of the distribution, but also on the frequency and positioning of the points representing that shape.[7] Furthermore, with engineering applications again in view, one may like to apply the above formula a) for not-so-smooth and/or not-so-slender configurations and b) over the transonic regime even in cases where $\mathcal{S}'(L)\neq 0$. In such applications, the above-mentioned numerical procedures present some difficulties. The disadvantage of Eminton's procedure[3] is that it forces $\mathcal{S}'(L)$ to zero and further it involves matrix inversion and converges slowly and is suspected to lose accuracy as the number of input points goes up. The procedure of Shanbhag and Narasimha[4] is prone to large errors since it makes use of only a small number (at most 10) of $S''(x)$ values in evaluating the integral with a logarithmic singularity. These $S''(x)$ values will have to be obtained numerically from the given discrete set of $S(x)$ and these are the sources of large errors in the scheme of Shanbhag and Narasimha. Spline techniques may suggest themselves in getting improved values of $S''(x)$ but the necessity of having to provide end conditions beyond mere point values makes these Spline formulations no better than other simpler ad hoc interpolation shcemes.[7]

A method is given here that requires only the first derivative values for evaluating the double integral and is consequently insensitive to the numerical differentiation errors. This method is far simpler than Eminton's, and is found to give reliable results in a variety of cases and takes less computer storage while taking comparable time.

Three examples are included for comparing the present results with the procedures of Eminton[3] and Shanbhag and Narasimha,[4,5] and with the wind-tunnel results of NAL for two of the examples.

Derivation of the Formulae

In the procedure described in this Note, we evaluate all three terms appearing in Eq. (1,1a). The integrals appearing on the right-hand side of Eq. (1,1a) are expressed as limits of sums in the Riemannian sense, of course taking due care of the logarithmic singularity. Consider first the double integral which is expressed as

$$I=\int_0^1 S''(\xi)F(\xi)d\xi \quad (2)$$

where

$$F(\xi_m)=\int_0^1 S''(\xi)\ln|\xi-\xi_m|d\xi \quad (3)$$

$$=\sum_{k=1}^{N}\int_{\xi k-}^{\xi k+} S''(\xi)\ln|\xi-\xi_m|d\xi \quad (4)$$

where the invertal of integration has been divided into N subintervals of length $\Delta\xi$ and $\xi_{k+}=\xi_k+\Delta\xi/2$ and $\xi_{k-}=\xi_k-\Delta\xi/2$. In each subinterval, $S''(\xi)$ is approximated by

$$S''(\xi)\doteq 1/\Delta\xi[S'(\xi_{k+})-S'(\xi_{k-})]\equiv\Delta S'_k/\Delta\xi \quad (5)$$

Then

$$\int_{\xi_{k-}}^{\xi_{k+}} S''(\xi)\ln|\xi-\xi_m|d\xi \doteq \begin{cases}\Delta S'_k\cdot\ln|\xi_k-\xi_m|, & k\neq m\\ \Delta S'_m\cdot(\ln(\Delta\xi/2)-1), & k=m\end{cases} \quad (6)$$

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Here the expression for $k=m$ is obtained by carrying out the integration of $\ln|\xi-\xi_m|$ over the subinterval. Using the approximation (5) again, we obtain finally

$$\int_0^1 S''(\xi)F(\xi)d\xi=\sum_{i=1}^{N}[S'(i+1)-S'(i)]$$

$$\left\{\sum_{p=1}^{N}[S'(p+1)-S'(p)]\cdot\ln(|i-p|\Delta\xi)+\ln(\Delta\xi/2)-1\right\} \quad (7)$$

For computing, it is expedient to write this as

$$I=\lim_{\substack{N\to\infty\\ \Delta\xi\to 0}}\sum_{i=1}^{N}\left\{[S'(i+1)-S'(i)]\sum_{\substack{p=1\\ p\neq i}}^{N}[(S'(p+1)-S'(p))\right.$$

$$\left.\cdot\ln(|i-p|\cdot\Delta\xi)]+[\ln(\Delta\xi/2)-1][S'(i+1)-S'(i)]\right\} \quad (8)$$

Finally, proceeding in a similar manner the single integral on the right-hand side of Eq. (1a) can be written as

$$\int_0^1 S''(\xi)\ln(1-\xi)\mathrm{d}\xi\equiv F(1)$$

$$=\sum_{k=1}^{N-1}\{[S'(k+1)-S'(k)]\cdot\ln[(N-k+\tfrac{1}{2})\Delta\xi]\}$$

$$+[S'(1)-S'(1-\Delta\xi)]\cdot\ln(\Delta\xi) \quad (9)$$

Numerical Procedure

From the given tabular values of ξ and $S(\xi)$ the first derivatives that are required while using the schemes given in Eqs. (8) and (9) for evaluating the integrals occuring in Eqs. (1) and (1a), can be obtained by any standard numerical differentiation procedure. In the present study 4-, 5-, and 6-point Lagrangian interpolation formulae were used.

Starting with a value of $N=2$, the number of subintervals of (0,1) were doubled successively. It was found that for most cases $N=256$ was large enough to yield a converged value.

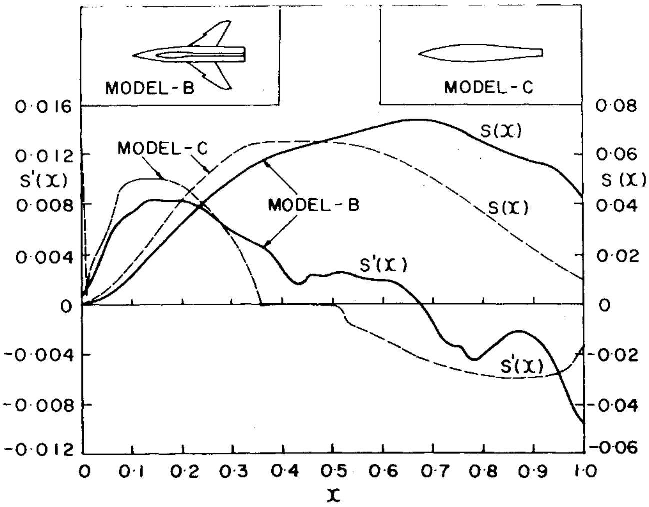

Fig. 1 *S(x)* and *S′(x)* curves for the two examples.

Examples Studied

The scheme presented here has been applied to three configurations referred to here as Examples A, B, and C. Example A is that suggested by Eminton [3,4] where $S(\xi)$ is given analytically by a polynomial with $S(1)=0$. Example B refers to a typical aircraft configuration with $S(\xi)$ and $S''(\xi)$ continuous and $S'(1)\neq 0$ (see Fig. 1). Example C refers to a typical bomb model with $S'(\xi)$ discontinuous and also $S'(0)\neq 0$ and $S'(1)\neq 0$ (see Fig. 1). Even though the linear theory is not valid for cases where $S'(0)\neq 0$, it has deliberately been applied to Example C [(constraining $S'(0)=0$] with a view to examining whether the results then obtained can give reasonable estimates for engineering purposes.

Models corresponding to Examples B and C have been tested in the transonic regime in the NAL 4′ Trisonic Wind Tunnel and an equivalent body of revolution of Example B has been tested in the NAL 1′ tunnel in the transonic regime and the results are presented in Fig. 2.

Table 1 Comparative study (Mach-dependent term is evaluated for $M=1.02$)

		Example A (Exact $C_{DW}=127.961$)			Example B			Example C		
Method	Lagrangan used (pt)	Double integral alone	T (rest of the terms Eq. 12)	C_{DW}	Double integral alone	T (rest of the terms Eq. 12)	C_{DW}	Double integral alone	T (rest of the terms Eq. 12)	C_{DW}
Present	4	123.898	2.359	126.257	0.02209	−0.00424	0.01785	0.16030	−0.02388	0.18418
	5	127.309	0.246	127.555	0.02114	−0.00352	0.01762	0.16602	−0.02344	0.18946
	6	127.940	−0.219	127.721	0.01989	−0.00281	0.01708	0.16952	−0.02333	0.19284
Shanbhag	4	122.422	2.649	125.071	0.01921	−0.00535	0.01386			
Narasimha	5	128.715	0.245	128.960	0.01802	−0.00747	0.01055			
	6	128.304	−0.220	128.084	0.01707	−0.00451	0.01256			
Eminton (64 intervals)		−128.282	0	128.282	0.02595	0	0.02595	0.22742	0	0.22742

Table 2 Comparison of convergence rates for the double integral between the present scheme and Eminton's procedure

Method	Example	2	4	8	16	32	64	128	256
Present	A	4.904	4.803	92.758	119.991	125.224	126.532	127.038	127.309
	B	0.01120	0.01573	0.01987	0.02023	0.02068	0.02093	0.02108	0.02114
	C	0.04761	0.09490	0.13015	0.13528	0.15231	0.16314	0.16523	0.16602
Eminton	A	15.460	109.920	119.984	124.906	127.205	128.282		
	B	0.00839	0.01206	0.01356	0.01970	0.02300	0.02595		
	C	0.14011	0.19727	0.20040	0.21361	0.21941	0.22742		

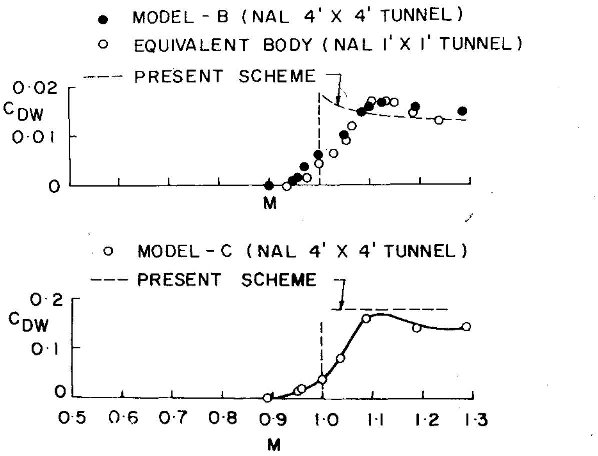

Fig. 2 Comparison of wave drag values by the present scheme with the experimental results of NAL-Model B and Model C.

Discussion of Results

The results obtained by the present scheme for these examples are compared with those by the schemes of Eminton[3,4] and Shanbhag and Narasimha[5,6] (see Table 1).

In Table 1 the value of the double integral (1) and the contribution T from the terms of Eq. (1a) are shown separately. In the scheme due to Eminton, T is exactly zero in all case since $S'(1)$ is forced to be zero. In Example A, $S'(1)=0$ exactly but $T\neq 0$ by the present scheme, as well as by that of Shanbhag and Narasimha due to numerical differentiation errors. However, as can be seen from Table 1 the contribution from T has a compensating effect on the final value in the present method. Furthermore, taking the contribution of the extra terms seems to reduce the sensitivity of the final value to the numerical interpolation schemes used in obtaining the derivative values.

In the Examples B and C, $S'(1)\neq 0$ and therefore $T\neq 0$, except in the case of Eminton's scheme as metnioned earlier. The comparison with the experimental results (Figs. 2) seems to indicate, that in the present scheme the contribution from T has again a desirable effect on the drag value. Furthermore, to some extent, the sensitivity to interpolating schemes is again found to be reduced by taking into account the contribution of T. This does not seem to be the case with the scheme of Shanbhag and Narasimha.

It may be noted here that for the case of Example B the double integral alone by the scheme of Shanbhag and Narasimha is closer to the maximum of the experimentally observed wave drag in the transonic regime. Consideration of the contribution due to T gives here a considerably lower value than the experimental values. In the case of Example C where discontinuities exist in the $S'(\xi)$, numerical valuse of $S''(\xi)$ around these discontinuities are not realistic and these result in unrealistic values of wave drag by the scheme of Shanbhag and Narasimha, therefore, these drag values have been omitted from Table 1.

The comparison of the present results with the experimental results shown in Fig. 2 shows that, apart from the reliability of the present scheme in the transonic regime, the agreement is very good for $M>1.20$. This shows that the present scheme would be quite useful in conjunction with supersonic area rule. Computation times on the IBM 360-44 machine were small and comparable for all the methods tried here.

In conclusion, the present simple method, requiring, as it does, only the first-derivative values of the area distribution, seems to be more reliable than the rest of the methods known in literature for evaluating the transonic wave drag.

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Static Aeroelastic Twist Effects on Helicopter Rotor-Induced Velocity

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Introduction

UNLIKE fixed-wing aircraft, the unsteady aerodynamic flowfield associated with a helicopter rotor, in hover or vertical flight, at low inflow conditions is indeed complicated. This is mainly due to the close proximity of the unsteady wake vortices beneath the rotor. In such a case, the use of fixed-wing unsteady aerodynamic theory in the flutter and dynamic response analyses is highly questionable. Unsteady aerodynamic theories have been developed[1-3] to account for the presence of wake beneath the rotor. To compute the unsteady airloads using these theories, the required wake spacing, h, beneath the rotor, is generally computed by using the combined blade element-momentum theory,[4] assuming the blade to be rigid. However, the rotor blade is flexible, both in torsion and bending, and, hence, there will be a change in the mean induced velocity distribution due to the static elastic twist of the blade. For a more realistic wake spacing, the effect of static elastic twist should be considered. In this Note, an approximate analytical treatment is given to determine the static elastic twist and the mean inflow velocity distribution. This inflow velocity can then be used to compute the wake spacing required for determining the unsteady airloads.

Analysis

Consider an elastic uniform untwisted blade in hover or vertical flight V, fixed at a distance a outboard from the axis of rotation and set at a pitch angle α_r. The actual blade is elastic both in torsion and bending, and hence it deforms under steady aerodynamic loads. However, in the present analysis, the edgewise bending stiffness is assumed to be large. Also, the effect of flapwise bending deformations on elastic twist θ is small in practice and hence one need only consider the torsional equilibrium equation in the analysis. The equilibrium equation in torsion[5] is

$$-(\mathrm{d}/\mathrm{d}r)\{[1+(Tk_A^2/GJ)](\mathrm{d}\theta/\mathrm{d}r)\}+K_1\theta=(M(r)/GJ)-K_2 \quad (1)$$

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